

This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

**Usage guidelines**

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

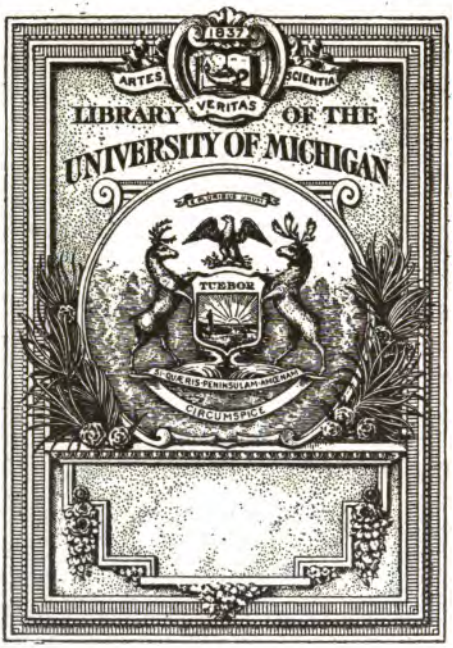
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

**About Google Book Search**

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`




THE GIFT OF
W. H. Butts

TA
562
.L265

# ADJUSTMENTS

OF THE

# COMPASS, TRANSIT, AND LEVEL.

BY

A. V. LANE, C.E., PH.D.,

ASSOCIATE PROFESSOR OF MATHEMATICS,
UNIVERSITY OF TEXAS.

BOSTON:
PUBLISHED BY GINN & COMPANY.
1886.

W. H. Butts
gt.
4-17-1923

Entered, according to Act of Congress, in the year 1886, by
A. V. LANE,
in the Office of the Librarian of Congress, at Washington.

J. S. Cushing & Co., Printers, Boston.

E 7-7-26 adv.

# PREFACE.

AN examination of those text-books in which this subject is or should be treated, reveals the fact that it is for the most part very meagrely and arbitrarily presented, sometimes dismissed with the statement that the adjustments should be made by the maker of the instrument. The method, when given, is generally without explanation or proof that it will accomplish the desired object, so that the student must either take the author's word for it, get some one to explain it to him, or work it out for himself; and being usually unprepared for such original work, he is in danger of adopting the first course mentioned, or of leaving the matter in doubt and mystery.

A great source of trouble lies in the fact that such authors are expected to express themselves in accurate terms and do not; the word "half," for example, being so often used for that which is at best but approximately so, that the student marvels at the talisman for such diverse operations being so uniformly that particular fraction.

Perhaps the absence of explanation and proofs of the

421818

correctness of some of the methods is largely due to the difficulty of making the subject clear to those who have not studied Descriptive Geometry. It is believed, however, that one whose attainments in the line of mathematics go no further than through Elementary Trigonometry will experience no difficulty with the following discussion of the adjustments of the three principal instruments used by surveyors and engineers.

This little volume has been called forth by the need of such an exposition of the subject, felt by the author for some time past in presenting the matter to his classes in Engineering, and any suggestions in the line of improvement will be acceptable.

A. V. LANE.

UNIVERSITY OF TEXAS,
AUSTIN, May, 1886.

## ABBREVIATIONS.

*A*, horizontal axis of telescope (Transit).
*B*, axis of the level bar (Level).
*C*, line of collimation (Transit, Level).
*H* and *V*, horizontal and vertical plane of reference.
*I*, intersection of *A* and *C* (Transit).
*R*, plane of revolution of adjusted *C* (Transit).
*S*, axis of spindle (Compass, Transit, Level).
*W*, intersection of cross-wires (Transit, Level).
*Y* and *y*, longer and shorter distances from *B* to the centers of the telescope's wye bearings (Level).
*l.t.*, level tube (Compass, Transit, Level).
*l.t.c.*, level tube-case (Compass, Transit, Level).
*l.t.c.a.*, level tube-case axis (Compass, Transit, Level).

# I. THE COMPASS.

**1. The Plate-Levels,** — *to so adjust them that when their bubbles are centered, the plate shall be horizontal.*

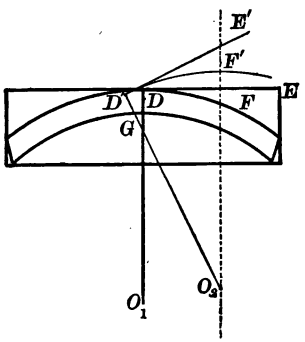



FIG. 1.

Let the *l.t.c.* $GE$ be turned in a vertical plane and about its center $G$ through an angle. The bubble, which was at $D$, the center of the *l.t.*, will move to a point $F'$ found by raising a vertical through the new position of the center of curvature of the *l.t.* The arc $D'F'$ through which the bubble has moved subtends the angle

$O_2 = DGD' =$ the angle between $DE$ and $D'E'$;

*i.e.*, the motion of the bubble is proportional to the angle through which the *l.t.c.* is turned about its center.

Let us suppose that the *l.t.c.a.* $D_1E_1$ is not parallel to the plate $JK_1$, but makes with it an angle $a$. Then, when the bubble is brought to the center, $D_1E_1$ being horizontal, $JK_1$ will differ $a$ from horizontality. If now the plate

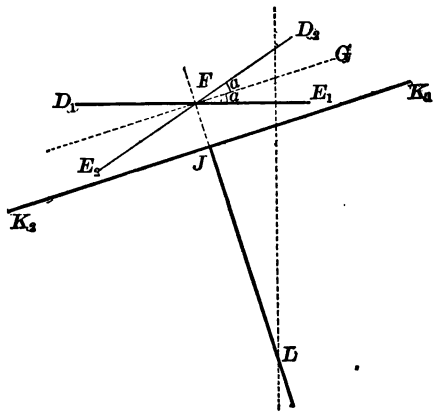



FIG. 2.

be turned 180° on $S$, the *l.t.c.a.* will have changed ends, and will lie in a vertical plane parallel to the one which before contained it, and distant therefrom by twice the distance of the *l.t.c.a.* from $S$. Since the two halves of the *l.t.c.* are precisely alike, the effect of this operation on the motion of the bubble is clearly the same as that of a rotation in a vertical plane and about the center of the *l.t.c.* through an angle $2a$. If then the bubble is

brought half-way back to the center by the screws at the ends of the *l.t.c.*, and the rest of the way by changing the position of *S*, the *l.t.c.a.* will take the adjusted position *FG* (parallel to the plate), and *S* will be vertical, the plate also horizontal. See Methods of Adjustment, p. 33.

**2. The Sights,** — *to make them vertical when the plate is horizontal.*

The plate being horizontal, if a vertical line is sighted to and each sight made to range with it from any point of the other, they will be vertical. See Methods.

**3. The Needle,** — *to so adjust it that any one vertical plane may contain its end points, its center of rotation, and the center of the graduated circle.*

If the ends of the needle do not in every position cut opposite degrees (point to divisions which differ 180°), this adjustment is needed. Failure to do so is due to the point of the pivot not being in a perpendicular to the plane of the graduated circle at its center, or to the needle being bent, or to both causes.

Suppose that, one end of the needle pointing to a division $G_1$, the other fails to point to its opposite, $J_1$. Let the pivot be bent until the failure is corrected, when the conditions will be such as are represented in the diagram. Reversing the sights exactly, the points $F_1$, $D_1$, $E_1$, $E_1'$ take the positions $F_2$, $D_2$, $E_2$, $E_2'$. Evidently the divis-

ion $N_2$ is approximately half-way between $M_2$ and $G_2$, the angle $F_2D_2O$ differing slightly from $F_1D_1O$, owing to the displacement of $F_1$ and $F_2$ from $O$. Hence, if the needle be so bent that its end point $E_2$ cuts half-way back to $G_2$, its ends will be approximately in line with its center. If the pivot be now bent until the needle cuts the readings $J_2$ and $G_2$, $F_2$ will be at or very near some

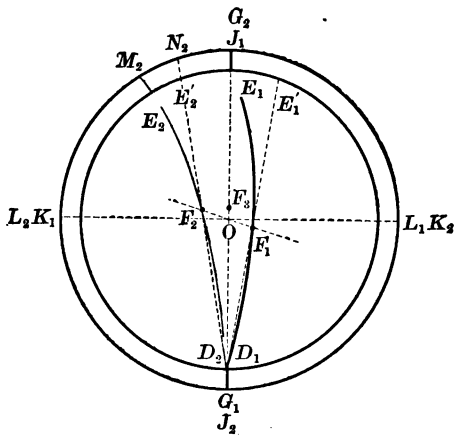



FIG. 3.

point as $F_3$ in the line $G_2J_2$. So by repeating this process the ends of the needle will be brought into line with its center, which center will be somewhere in the line $G_2J_2$, as at $F_3$.

When this has been done, if one end of the needle be held at $K_2$ (90° from $J_2$) and the pivot bent towards

$K_2L_2$ until the other end cuts $L_2$, $F_3$ will be brought into the line $K_2L_2$ and nearer to its proper position.

By repeating this last operation on the pivot with reference to the lines $G_2J_2$ and $K_2L_2$ alternately, its point will be adjusted.

# II. THE TRANSIT.

**1. The Levels.** — Same as I. 1.

**2. The Needle.** — Same as I. 3.

**3. The Line of Collimation,** — *to make it perpendicular to the horizontal axis of the telescope, so that, by revolution about that axis, it will generate a plane and not a conical surface.*

It should be remembered that $C$ is the line of sight and is determined by the optical center of the object-glass and the intersection of the cross-wires. The wires should be respectively vertical and horizontal; they may be made so by causing one to range with a plumb-line or known vertical line, since they are perpendicular to each other.

Assume a horizontal circle with center $I$, and let $C_1$ differ $\alpha$ from $IE$, a perpendicular to $A$. Revolving $C_1$ about $A_1$ till it again cuts the circle, it takes the position $C_2$, differing $\alpha$ from $ID$, and therefore $180° - 2\alpha$ from $C_1$. Turning the instrument about $S$ $180° - 2\alpha$, $C_2$ becomes $C_3$ in coincidence with $C_1$, while $A_2$ takes the position $A_3$. Revolving $C_3$ about $A_3$ until it again cuts the circle, it takes the position $C_4$, differing $\alpha$ from $C_5$ perpendicular

to $A_5$. $C_5$ and $ID$, respectively perpendicular to $A_5$ and $A_2$, differ by the same angle as those positions of the axis; viz., $2\alpha$. Since the arc 4 5 subtends $\alpha$, $5D$ subtends $2\alpha$, and $D2$ subtends $\alpha$, the ratio of the arc 4 5 to the arc 4 2 is clearly that of $\alpha$ to $4\alpha$, or one-fourth. If then $C_4$ were at $C_5$, one-fourth of the way toward $C_2$, it would

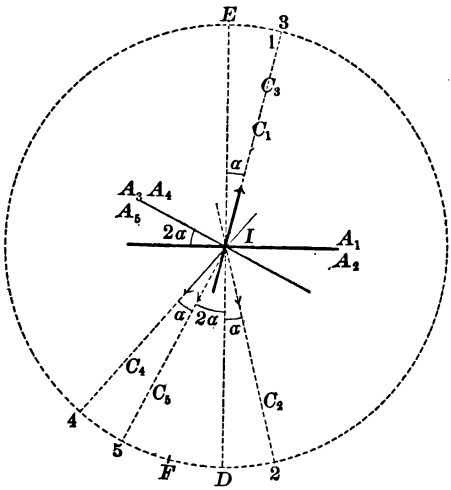



FIG. 4.

be perpendicular to the horizontal axis of the telescope, and hence in adjustment. In practice the above conditions cannot usually be perfectly realized. Thus the points 1, 2, 4, 5 are not in one plane, which introduces a slight error, from the fact that, until adjusted, $C$ revolves about $A$ in a conical surface and so moves to the right or left as it leaves a horizontal position; and again, the

points 2, 4, 5 are not taken exactly on a circle, but in a straight line on top of a stake. The errors in each case are small, as the displacements of the points from a horizontal plane and the distances apart of the last three are taken quite small in comparison with their distance from $I$.

After this has been done, it may be found that the intersection of the cross-wires is not in the center of the field of view of the eye-piece. Since the position of $C$ depends only on the object-glass and $W$, we may, without affecting it, move the eye-piece by the proper screws until the center of the field of view is brought into coincidence with the intersection of the cross-wires. For the eye-piece is simply a microscope with which we magnify and invert the inverted image formed at the cross-wires where its focus is.

Again, it is evident that, even when $C$ is not perpendicular to $A$, we may locate three points on the same side of the instrument, which will either be in one vertical plane (in line), if they are all at the same height as $I$; or very nearly in line, if they do not vary much from that height. Let these points be 1, $I$ and $F$ in the preceding diagram. Setting up the instrument at the middle one $I$, sighting to 1, and turning $C_1$ about $A_1$, it comes to $C_2$. If $C_2$ were perpendicular to $A_2$, the point $D$ would be sighted to. So the point 2 is twice as far from $F$ as the point $D$ upon which to effect adjustment.

**4. The Standards,** — *to make the bearings of the horizontal axis of the telescope equally distant from the plate, so that when the instrument is levelled, the line of collimation will, by revolution about the horizontal axis of the telescope, generate a vertical plane.*

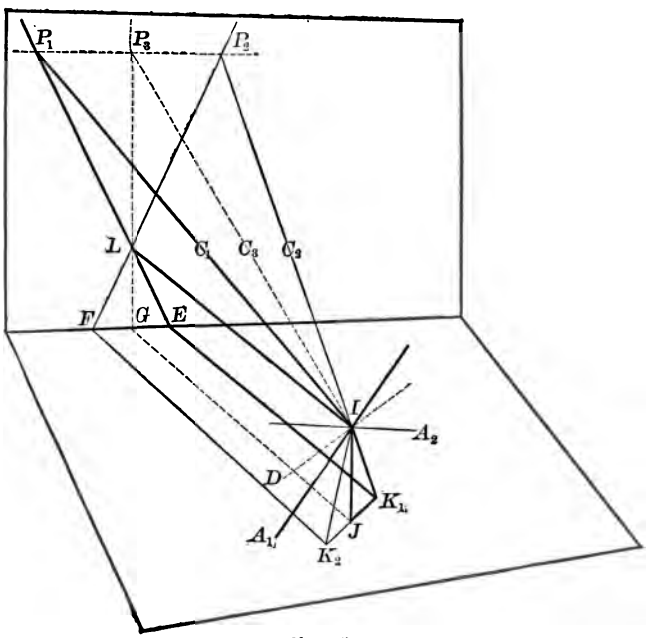



FIG. 5.

Suppose the instrument set up on $H$, level, and with the preceding adjustments made. Suppose $A$ horizontal and occupying a position $ID$ parallel to $H$, but not necessarily so to $V$. Draw the vertical $IJ$ and, through $J$, the line

$K_1K_2$ in $H$ parallel to $ID$. Through $J$ draw $JG$ perpendicular to $K_1K_2$; and, through $G$, $GP_3$ perpendicular to the intersection of $H$ and $V$. Now $R$, being perpendicular to $A$ or $ID$, is perpendicular to $H$, and must cut $V$ in a line perpendicular to the intersection of $H$ and $V$. Hence $R$ must contain $IJ$, $JG$, and $GP_3$. Take $L$ in $GP_3$ so that $GL = JG$. $C$ may be directed to $L$, when it will be parallel to $H$ and perpendicular to the plane $DIJ$. Since the arm $DI$ of the right angle $DIL$ may be rotated about $IL$ into the position $A_1$, $A$ may take the position $A_1$ in the plane $DIJ$, and $C$ be still fixed upon $L$. Since, if turned into that plane, $C$ will take the position $IK_1$, perpendicular to $A_1$, $R$, containing $K_1$ and $IL$, must now cut $H$ in a line $K_1E$ parallel to $IL$ and therefore to $JG$, and so must cut $V$ in $EL$.

If now the instrument be turned on $S$ or $IJ$ through 180°, $A_1$ takes a position $A_2$, making the same angle with $IJ$ as before. $A_2$ being in the plane $DIJ$, $C$ may still be directed to $L$ or, in that plane, to $K_2$, $JK_2$ being equal to $JK_1$, and the triangle $JIK_1$ having turned into the position $JIK_2$. $R$ must therefore now cut $H$ in a line $K_2F$, parallel to $JG$, and $V$ in a line $FL$.

Draw $P_1P_2$ in $V$ and parallel to the intersection of $H$ and $V$. With the axis in the position $A_1$, $C$ could be directed to $P_1$; with the axis in the position $A_2$, to $P_2$; and in the adjusted position $DI$, to $P_3$. The wires should therefore be moved toward $P_1$ by an amount $P_2P_3$.

Since $JG$ bisects $K_1K_2$, it bisects $EF$, and therefore

$GL$ bisects $P_2P_1$. So the wires should be moved halfway from their second position, $P_2$, toward their first, $P_1$, by changing the height of one of the standards.

Practically, the ground and a vertical wall take the place of $H$ and $V$. (Given last, under Methods, II. 4.)

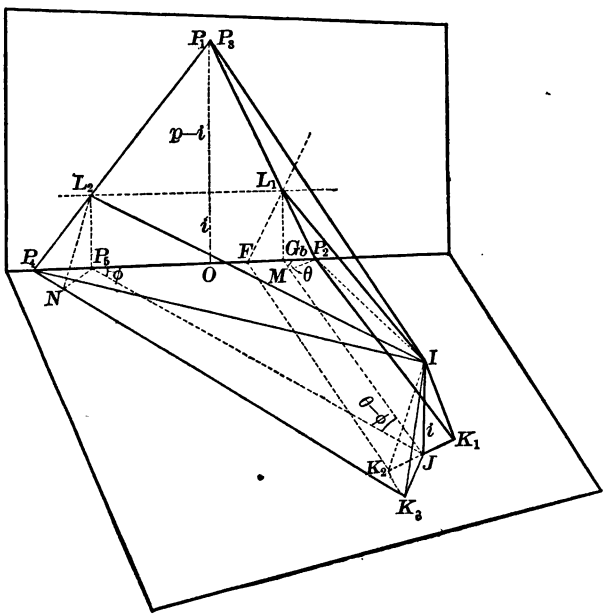



FIG. 6.

Suppose the instrument at $IJ$, the right-hand end of $A$ the higher, and $R$ occupying the position $P_1P_2K_1$. Imagine the instrument turned 180° on $S$, so that the triangle $IJK_1$ occupies the position $IJK_2$, and $R$ the position

$L_1FK_2$, just as in the preceding case. Directing $C$ to $L_1$, imagine it turned to the left, remaining horizontal, until a point $L_2$ is reached, such that, on revolving $C$ about $A$, it will again cut $P_1$. In order for $L_2$ to be such a point, it is evident that the triangle $IJK_2$ must have revolved into a position $IJK_3$, such that perpendiculars $JP_5$ and $K_3P_4$ to $JK_3$ shall meet the intersection of $H$ and $V$ in points $P_5$ and $P_4$, respectively on a vertical through $L_2$ and on the prolongation of $P_3L_2$.

$C$ was first turned down from $P_1$ to $P_2$, then from $P_3$ to $P_4$, $A$ lying in the plane $IJK_3$. If $A$ became horizontal in that plane, $R$ would occupy the position $L_2P_5J$. So the wires should be moved from $P_4$ to $P_5$, by changing the height of one of the standards. We desire therefore to determine the ratio of $P_4P_5$ to $P_4P_2$.

Refer the point $J$ to $P_2$ by $JG = d$, angle $JGP_2 = \theta$, $GP_2 = b$. Let $L_2P_5 = L_1G = IJ = i$. Let $P_1O = p$, and the angle $K_3P_4P_5 =$ angle $JP_5G = \phi$. Then the angle $GJP_5 = \theta - \phi$; also

$$P_5N = JK_3 = JK_2 = JK_1 = MP_2 = b \sin\theta.$$

By similar triangles

$$\frac{P_4P_5}{P_4O} = \frac{i}{p} = \frac{P_2G}{P_2O}. \qquad (1)$$

By alternation

$$\frac{P_4P_5}{P_2G} = \frac{P_4O}{P_2O},$$

and this, by composition, gives

$$\frac{P_4P_5 + P_2G}{P_2G} = \frac{P_4O + P_2O}{P_2O},$$

or

$$\frac{P_4P_5 + b}{P_2G} = P_4P_2 \cdot \frac{1}{P_2O} = P_4P_2 \cdot \frac{i}{p \cdot P_2G}, \quad \text{by (1).}$$

Whence

$$P_4P_5 = P_4P_2 \cdot \frac{i}{p} - b. \tag{2}$$

In the triangle $NP_4P_5$ we have

$$\tan\phi = \frac{b\sin\theta}{\sqrt{P_4P_5^2 - b^2\sin^2\theta}}; \tag{3}$$

and in the triangle $P_5GJ$

$$\frac{P_5G}{d} = \frac{\sin(\theta - \phi)}{\sin\phi} = \frac{\sin\theta}{\tan\phi} - \cos\theta;$$

whence, by (3),

$$\frac{P_5G}{d} + \cos\theta = \frac{\sqrt{P_4P_5^2 - b^2\sin^2\theta}}{b}. \tag{4}$$

$$P_5G = P_4P_2 - (P_4P_5 + b) = P_4P_2 \cdot \frac{p - i}{p},$$

and

$$P_4P_5 = \frac{P_4P_2 \cdot i - bp}{p}, \quad \text{by (2).}$$

Substituting these in (4) and clearing of fractions, we have

$$b \cdot P_4P_2(p - i) + bdp\cos\theta$$
$$= d\sqrt{P_4P_2^2 \cdot i^2 - 2bip \cdot P_4P_2 + b^2p^2 - b^2p^2\sin^2\theta}.$$

Squaring, cancelling, and taking out the common factor $P_4P_2$, we find

$$P_4P_2 = \frac{2bdp\,[di + b\,(p-i)\cos\theta]}{d^2i^2 - b^2\,(p-i)^2}.$$

Substituting this in (2), we obtain

$$\frac{P_4P_5}{P_4P_2} = \frac{i}{p} - \frac{d^2i^2 - b^2(p-i)^2}{2dp\,[di + b\,(p-i)\cos\theta]},$$

the required ratio.

If $\theta$ be 90° and $p - i = d$, this ratio becomes

$$\frac{i}{p} - \frac{i^2 - b^2}{2ip},$$

or, neglecting the small fraction $\frac{b^2}{2ip}$, it becomes $\frac{i}{2p}$.

These conditions are usually very nearly satisfied, so we may consider $\frac{i}{2p}$ quite an approximate value of the true ratio. Assuming $i = 5$ ft. and $p = 50$ ft. as about average values, this ratio equals $\frac{1}{20}$; while, even if $b$ could equal 6 inches, the neglected fraction $\frac{b^2}{2ip}$ would only equal $\frac{1}{2000}$. (Given second, under Methods, II. 4.)

Suppose that, having as before located the point $P_2$ from the point $P_1$, we turn the instrument about halfway around on $S$ and, revolving the telescope, fix the wires again on the lower point $P_2$. Now, raising them

to $P_4$ at height of $P_1$, we wish to determine the ratio of $P_4P_5$ to $P_4P_1$.

For, as before, $R$ having first the position $P_1P_2K_1$, on turning exactly half-way around, it would take the position $L_1FK_2$; and, directing $C$ to $L_1$, we must turn the

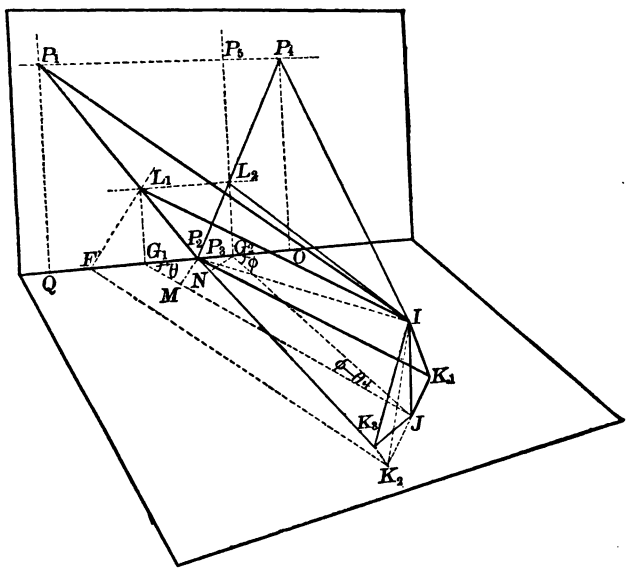



FIG. 7.

instrument to the right until $C$ strikes a point $L_2$, such that, on revolving about $A$, the point $P_2$ is again cut.

The triangle $IJK_2$ has now revolved into the position $IJK_3$, and $R$ into the position $P_4P_3K_3$; while, if $A$ became horizontal, $R$ would have the position $P_5G_2J$.

The references of $J$ to $P_2$ are now $JG_1 = d$, angle $JG_1P_2 = \theta$, $G_1P_2 = b$.

The angle $JG_2O$ is $\phi$, and the angle $G_1JG_2 = \phi - \theta$.

Since, by similar triangles,

$$\frac{OG_2}{P_3G_2} = \frac{p-i}{i}, \quad \frac{P_4P_5}{P_4P_1} = \frac{OG_2}{OQ} = \frac{p-i}{i} \cdot \frac{P_3G_2}{OQ};$$

and we will first determine $\frac{P_3G_2}{OQ}$.

$$\frac{P_3G_2}{OP_3} = \frac{i}{p} = \frac{P_3G_1}{P_3Q}. \tag{1}$$

By alternation,

$$\frac{P_3G_2}{P_3G_1} = \frac{P_3O}{P_3Q},$$

and this, by composition, gives

$$\frac{P_3G_2 + P_3G_1}{P_3G_1} = \frac{P_3O + P_3Q}{P_3Q},$$

or,

$$\frac{P_3G_2 + b}{P_3G_1} = OQ \cdot \frac{1}{P_3Q} = OQ \cdot \frac{i}{p \cdot P_3G_1}, \quad \text{by (1)}$$

Whence

$$P_3G_2 = OQ \cdot \frac{i}{p} - b. \tag{2}$$

In the triangle $NG_2P_3$ we have

$$\tan \phi = \frac{b \sin \theta}{\sqrt{\overline{P_3G_2}^2 - b^2 \sin^2 \theta}}, \tag{3}$$

and, in the triangle $G_1G_2J$,

$$\frac{G_2G_1}{d} = \frac{\sin(\phi-\theta)}{\sin\phi} = \cos\theta - \frac{\sin\theta}{\tan\phi};$$

whence, by (3),

$$\cos\theta - \frac{G_2G_1}{d} = \frac{\sqrt{P_3G_2^2 - b^2\sin^2\theta}}{b}. \qquad (4)$$

$$G_2G_1 = P_3G_2 + b = OQ\cdot\frac{i}{p},$$

and

$$P_3G_2 = \frac{OQ\cdot i - bp}{p}, \quad \text{by (2)}.$$

Substituting these in (4) and clearing of fractions, we have

$$bdp\cos\theta - bi\cdot OQ = d\sqrt{OQ^2\cdot i^2 - 2bip\cdot OQ + b^2p^2 - b^2p^2\sin^2\theta}.$$

Squaring, cancelling, and taking out the common factor $OQ$, we find

$$OQ = \frac{2bdp}{i}\cdot\frac{d - b\cos\theta}{d^2 - b^2}.$$

Substituting this in (2), we obtain

$$\frac{P_3G_2}{OQ} = \frac{i}{p} - \frac{i}{2dp}\cdot\frac{d^2-b^2}{d-b\cos\theta} = \frac{i}{p}\cdot\frac{d^2 - 2bd\cos\theta + b^2}{2d(d-b\cos\theta)}.$$

Therefore

$$\frac{P_4P_5}{P_4P_1} = \frac{p-i}{i}\cdot\frac{P_3G_2}{OQ} = \frac{p-i}{2dp}\cdot\frac{d^2-2bd\cos\theta+b^2}{d-b\cos\theta},$$

the required ratio.

If $\theta$ be 90°, neglecting the small fraction

$$\frac{(p-i)\,b^2}{2d^2p},$$

it becomes

$$\frac{p-i}{2p}=\frac{1}{2}-\frac{i}{p},$$

or approximately $\frac{1}{2}$, since $\frac{i}{2p}$ is usually about $\frac{1}{20}$; while the neglected fraction, under the preceding conditions, is $\frac{1}{18000}$. (Given first, under Methods, II. 4.)

This last is the best and most generally used method. The first is excellent, if divisions on the horizontal circle such as the zeros may be depended on as exactly opposite. In the second, the amount through which the wires are to be moved for correction is such a small part of a small distance that it is difficult of exact application.

5. *a.* **The Vertical Circle's Vernier Zero,** — *to so adjust it that when the vertical circle's zero is in line with it, the line of collimation shall be horizontal.*

If, instead of a full circle firmly attached to $A$, there is only an arc of a circle, which may be turned upon and clamped to it, as is the case in some instruments, this adjustment depends upon and is made after that of the level attached to the telescope. The method of making it in that case is explained further on (after 6).

Let us suppose that the vernier zero $Z_1$, is not in its proper position (vertically below the center of the vertical circle); so that, when the circle zero $z_1$ is made to coincide with it, $C$ has the position $C_1$, making the same angle with the horizontal $O_1D_1$ that $O_1z_1$ makes with the vertical $O_1F_1$.

Let $P_1$ be some point in $C_1$. Turning the instrument 180° on $S$, $C_1$, $P_1$, $z_1$, $Z_1$ move to $C_2$, $P_2$, $z_2$, $Z_2$. The

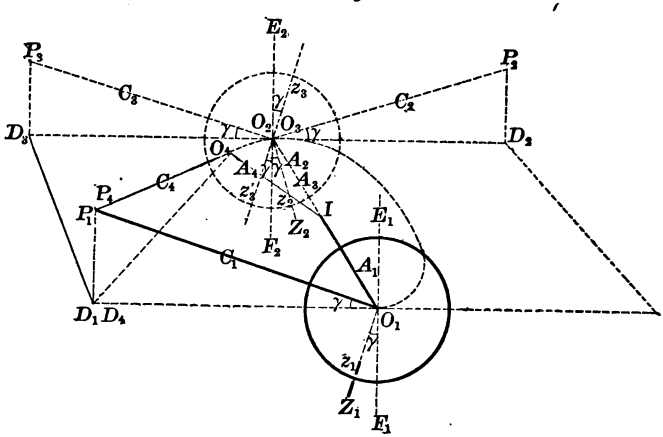



FIG. 8.

angle $P_2O_2D_2$ being equal to $\gamma$, if $C_2$ be turned on $A_2$ through $180° - 2\gamma$, it takes the position $C_3$, and $z_2$ the position $z_3$, the angles $P_3O_3D_3$ and $E_2O_3z_3$ being each equal to $\gamma$. The division $z_3'$ opposite $z_3$ is now $\gamma$ from $F_2$, and $2\gamma$ from $Z_2$. Hence if $Z_2$ were moved half-way to $z_3'$ and then $z_3'$ brought into coincidence with it, $C_3$ would

take the horizontal position $O_3D_3$, and, the zeros being also in coincidence, the adjustment would be effected. If $A_3$ were turned into the position $A_4$, and $C_3$ directed to $P_4$, the angle $P_4O_4D_4$ being slightly different from $\gamma$, the ratio corresponding to the above would not be exactly one-half, yet by taking the distance of $P_1$ from the instrument sufficiently great in comparison with $A$, the point $P_1$ may be used for $P_3$ with but little error; and it is so used, the process being repeated until the whole error becomes inappreciable.

**6.** *a.* **The Level attached to the Telescope,** — *to so adjust it that when its bubble is at the center, the line of collimation shall be horizontal.*

A full vertical circle being present, and its zero in coincidence with the vernier zero after the preceding adjustment, if the bubble is brought to the center by means of the nut at either end of the level, the adjustment is effected.

**5.** *b.* **The Level attached to the Telescope.** — If, however, the instrument has the movable arc, the level is first adjusted as follows:

Let $D$, $J_1$, $E$, and $J_3$ be four points in line, and in the order named; also equally distant horizontally. Suppose the instrument set up at $J_1$, and $C_1$ directed to a graduated rod held on the stake $D$, giving a reading $F_1D$. Turning the instrument 180° about $S$, without otherwise disturbing

$C_1$, $C_2$ will cut the rod held on the stake $E$ at a reading $F_2E$, the points $F_1$, $F_2$ being at the same height, since $C_1$ and $C_2$ make equal angles with the vertical $I_1J_1$, and

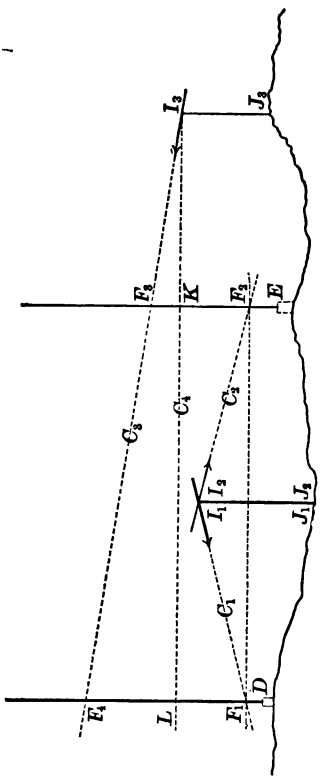



FIG. 9.

the positions of the rod are equidistant from $I_1$. Let the instrument be now set up at $I_3$, and, with $C$ in a posi-

tion $C_3$, the readings $F_3E$ and $F_4D$ noted. Draw $I_3KL$ parallel to $F_2F_1$, and therefore horizontal.

$F_4F_1 = F_4D - F_1D$ and $F_3F_2 = F_3E - F_2E$ being known, let $x = LF_1 = KF_2$, and we have, by similar triangles,

$$\frac{F_4F_1 - x}{F_3F_2 - x} = \frac{LI_3}{KI_3} = \frac{3}{1};$$

whence

$$x = \frac{3F_3F_2 - F_4F_1}{2}$$

is known.

So, by directing $C$ to a reading $DF_1 + x$, it takes the horizontal position $C_4$, and the level on the telescope may be adjusted by bringing its bubble to the center, by means of the nut at either end.

**6. *b*. The Vertical Arc's Vernier Zero.** — The vertical arc may now be turned on $A$ and clamped with its zero in coincidence with the index zero, effecting its adjustment.

For convenience of application as to sign, etc., it is well to take the first reading just above the top of the higher of the stakes $D$, $E$, and to set up at $J_3$ so that $I_3$ shall be higher than $F_1$, $F_2$; taking the readings $F_3E$, $F_4D$ greater than $F_2E$, $F_1D$ respectively.

By driving the stakes at $D$ and $E$ so that their tops are at $F_1$ and $F_2$, the first two readings reduce to zero, and $F_3F_2$, $F_4F_1$ are the only readings taken; they are used as stated above, $x$ being itself the reading of the target at $F_1$ for horizontality.

# III. THE LEVEL.

**1. The Line of Collimation,** — *to so adjust it that it will pass through the centers of the circles of the wye bearings of the telescope.*

This adjustment is a first step toward making $C$ perpendicular to $S$.

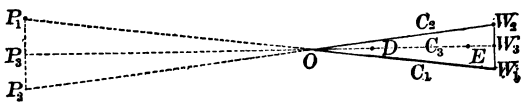



FIG. 10.

Let $D$ and $E$ be the centers of the circles of the wye bearings and $C_1$ the position of $C$ sighting to some point as $P_1$, distant $P_1P_3$ from the line $ED$ produced. If now $C_1$ be turned 180° about $ED$, it will take the position $C_2$, making with $ED$ an angle equal to that made by $C_1$, and sighting to some point as $P_2$ at the same distance as $P_1$ from $O$ and $P_3$. Clearly now for $C_2$ to coincide with $ED$, $W_2$ must be brought to $W_3$, half-way back to $W_1$, its former position, by moving it until $C_2$ takes the position $C_3$ sighting to a point $P_3$, half-way from $P_2$ towards $P_1$. This may be best accomplished by adjusting each wire in succession, if they be much in error, by means of some

line sighted to, until each is nearly right, then completing the adjustment of each. This amounts to substituting for $P_1$ a line through $P_1$ and perpendicular to the plane of the paper, for $P_2$ a like but imaginary line, and for $W_1$, $W_2$, and $W_3$ one of the cross-wires, also perpendicular to the plane of the paper and through those points.

As already explained in II. 3, the center of the field of view should now be brought into coincidence with $W$ by moving the proper screws, and the correctness of the centering may be tested by turning the telescope in the wyes, when the object should not appear to shift its position.

**2. The Level,** — *to make the level tube-case axis parallel to the line of collimation.*

In the explanation of I. 1 it was shown that in the operation of reversing the *l.t.c.a.* with reference to any plane or line to which its ends were referred, the bubble moves from the center in the first position to some point in the second position, such that the arc between the center and that point is double the arc through which the bubble should be moved back in order to make the *l.t.c.a.* parallel to the line or plane of reference. And this, too, whether that line or plane be horizontal or not.

Since now $C$ contains the centers of the circles of the wye bearings, by reversing the telescope end for end as to the wyes, we may effect this adjustment; the ends of the *l.t.c.a.* being referred to $C$. But there is a disturbing element to be considered in this connection.

The *l.t.c.* has a screw at one end for lateral adjustment, to bring the *l.t.c.a.* into one plane with *C*, and another, at the other end, for vertical adjustment to make these lines equidistant throughout. If we remove the clips from the wyes, and reverse the telescope end for end, it may be that we do not put it down with the vertical screw immediately below *C*, as before, but slightly rotated to one side. We must therefore examine the effect on the bubble of a slight rotation of the *l.t.c.a.* about *C*, with a view to ascertaining how much if any of the bubble's motion on reversal is due to this cause.

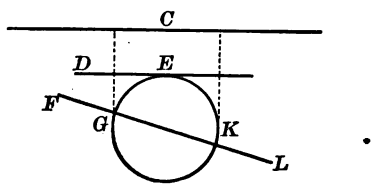



FIG. 11.

Imagine the plane of the paper vertical, and containing *C* and the *l.t.c.a.* Draw *DE* tangent to the *l.t.* at *E*. The highest point of the *l.t.* being that of contact with a horizontal tangent plane, it is evident that, *C* being horizontal, this highest point will lie on a cross-section of the *l.t.* at *E* for any revolution less than 90° about *C*, whether the *l.t.c.a.* is equidistant from *C* or not. For its position does not affect the symmetry of the parts of the *l.t.* on each side of *E*, but simply their amounts.

Suppose $C$ is not horizontal. After a revolution of 90°, $G_1G_2'$ and $K_1K_2'$ turning about $G_2'$ and $K_2'$ respectively, into a horizontal position, $G_1$ will be at the same height as $G_2'$, and therefore higher than $K_1$ will be (same height as $K_2'$). So the bubble will, as the rotation begins, start towards the end $G_1$.

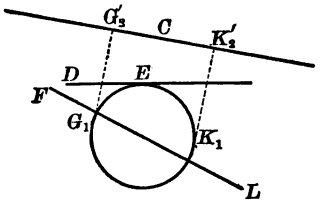



FIG. 12.

Again, in Fig. 11, suppose $L$ to be slightly in front of the plane of the paper, and $F$ a like amount behind. If we rotate towards us, $L$ rises and $F$ falls, the bubble therefore running toward $K$, instead of remaining at $E$. as before; while, if we rotate the other way, the reverse occurs. Also, in Fig. 12, if we make a like supposition, since the bubble before moved toward $G_1$, whichever way rotation took place, while the effect of this new supposition tends to make it move towards $K_1$ or $G_1$ (according as we rotate towards us or away), it is clear that these causes may combine to move the bubble one way, or may oppose and even neutralize each other.

Thus it is evident that, knowing nothing of the actual state of the positions of $C$ and the *l.t.c.a.*, we cannot

predicate anything as to how much of the motion of the bubble on reversal may be due to the cause just examined. Nevertheless the lateral screw may be, *by trial*, so adjusted as to eliminate the effect of this slight rotation, and finally to bring the *l.t.c.a.* into the same plane with $C$, as in the method given farther on.

**3. The Wyes,** — *to so adjust them, as to the distances of the centers of their circular bearings from the axis of the level-bar, that the axis of the spindle shall be perpendicular to the line of collimation.*

$B$ is supposed to be made perpendicular to $S$, so that if $Y$ be made equal to $y$, $C$ will be parallel to $B$, and therefore perpendicular to $S$. But if $B$ differs by some angle $\beta$ from perpendicularity to $S$, by reason of bad construction or some strain received, $Y$ and $y$ must be given such values as to counteract this and make $C$ perpendicular to $S$.

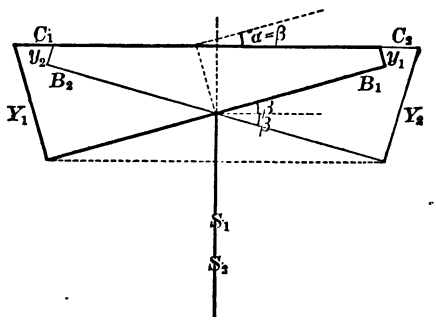



FIG. 13.

From the diagram it is easily seen that the necessary condition is $\alpha = \beta$, with opposing effects on the bubble (a special case of which is $\alpha = 0$, $\beta = 0$, when $C$ is parallel to $B$ as well as perpendicular to $S$), or, in other words, $C$ making the same angle with $B$ that $B$ does with a perpendicular to $S$, and $S$ making an acute angle with that half of $B$ which carries $Y$.

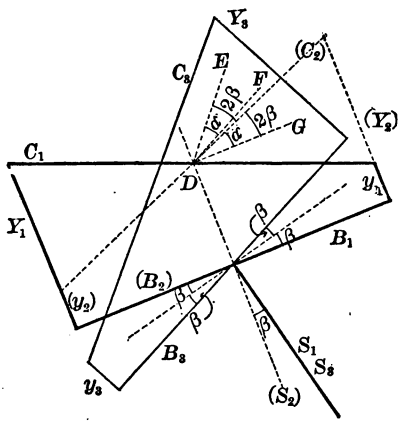



FIG. 14.

Let us suppose $S$ to have a position $S_1$, making an acute angle $90° - \beta$ with the $y$ half of $B$. We see that, by reversing about $S_1$, $B_1$ and $C_1$ take the positions $B_3$ and $C_3$; while if $S_1$ had the position $(S_2)$ perpendicular to $B_1$, $B_1$ and $C_1$ would take the positions $(B_2)$ and $(C_2)$, on reversal. Rotating $(B_2)$ through $2\beta$ into the position $B_3$, $(C_2)$ rotates through an equal angle into the position

$C_3$. Since, then, $C_3$ makes an angle $2\beta$ with $(C_2)$, and $(C_2)$ makes $2\alpha$ with $C_1$, $C_3$ makes $2\alpha + 2\beta$ with $C_1$.

This may also be shown thus: Drawing $DE$ parallel to $C_3$ and $DF$ parallel to $B_3$, we have $EDF = \alpha$. But $FDG = 2\beta$, since $FD$ is parallel to $B_3$ and $DG$ to $B_1$. Therefore the (acute) angle between $DF$ and $(C_2)$ equals $2\beta - \alpha$. Hence the (acute) angle between $DE$ and $(C_2)$ or $C_3$ and $(C_2)$ equals $\alpha + 2\beta - \alpha = 2\beta$; whence the (acute) angle between $C_3$ and $C_1$ equals $2\alpha + 2\beta$.

Thus we see that, on reversing about $S_1$, the bubble moves from the center toward $Y$ over an arc corresponding to $2\alpha + 2\beta$, the effects of $\alpha$ and $\beta$ being in this case cumulative. Remembering that only $2\alpha$ of this is due to $C$'s not being parallel to $B$, and that, if the bubble were moved back over $\alpha$, $C_3$ would be parallel to $B_3$, we see that if we shorten $Y$ until the bubble comes half-way back to the center, we move it over $\frac{1}{2}(2\alpha + 2\beta) = \alpha + \beta$; *i.e.*, $\beta$ more than the proper amount $(\alpha)$ to make $C_3$ parallel to $B_3$. Thus we have replaced the error $\alpha$ by another, $\beta$, and made that wye which was the longer, now the shorter; having decreased $Y$ too much for parallelism of $C_3$ and $B_3$. Since $Y$ and $y$ have interchanged ends, $S$ now makes an acute angle with the $Y$ half of $B$, and the angle between $C_3$ and $B_3$ is equal to that between $B$ and a perpendicular to $S$, — precisely the conditions of Fig. 13 (except that $S$ is not vertical). So that the adjustment is effected with $B$ not perpendicular to $S$.

Suppose the effects of $\alpha$ and $\beta$ on the bubble are oppos-

ing, that $\alpha$ is greater then $\beta$ ($S$ making an acute angle with the $Y$ half of $B$). Reversal about $S_1$ brings $B_1$ and $C_1$ into the positions $B_3$ and $C_3$; while reversal about ($S_2$) perpendicular to $B_1$ would have brought them into the positions ($B_2$) and ($C_2$). Introducing the effect of $\beta$,

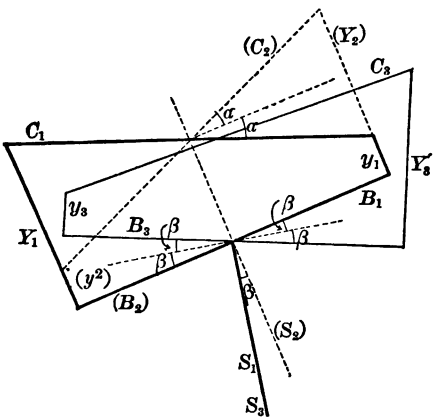



FIG. 15.

($C_2$), which had separated from $C_1$, returns toward but not to it, since $\alpha > \beta$. The combined effect of $\alpha$ and $\beta$ is thus to move the bubble over an arc corresponding to $2\alpha - 2\beta$ toward $Y_3$. Since, in bringing it half-way back to the center, it is moved over $\alpha - \beta$ ($\beta$ less than the proper amount to make $C_3$ parallel to $B_3$), there remains an error $\beta$ between $C$ and $B$. But since $Y_3$ has not been made shorter than $y_3$, $S$ is still making an acute angle with the $Y$ half of $B$, and so the adjustment is effected, as in Fig. 13.

Finally, suppose $a$ and $\beta$ opposing, and $\beta > a$. The effect of $a$ being to make the bubble run toward the ($Y_2$) end of ($C_2$), and $\beta$ causing ($C_2$) to turn back through $C_1$ to $C_3$ (making the bubble run away from the $Y_3$ end of $C_3$), their combined effect will move it $2\beta - 2a$ toward the $Y_3$ end of $C_3$. To move it half-way back we must now lengthen $Y_3$ (or shorten $y_3$), and so increase $a$ by $\frac{1}{2}(2\beta - 2a)$; *i.e.*, by $\beta - a$, making the angle between

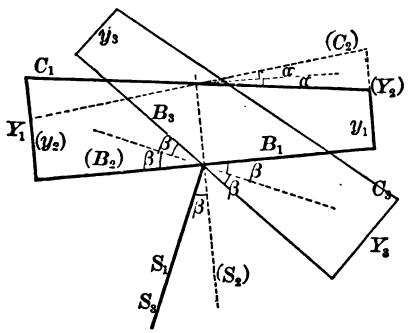



FIG. 16.

$B_3$ and $C_3$ $a + (\beta - a) = \beta$, $S$ still making an acute angle with the $Y$ half of $B$, — again the conditions of Fig. 13.

Thus we see that, when $a$ and $\beta$ are cumulative in effect, the process results in making them opposing, and if they are at first opposing, they result opposing. For, in the first case, we make the longer wye the shorter, thus introducing a change; while in the latter, although the length of one of the wyes is changed, the longer one

remains the longer. In each case $a$ is changed to the amount $\beta$ (there being no facilities for changing $\beta$), and so they are left equal and opposing.

It should be noted that, while $C$ is made perpendicular to $S$, by thus bringing the bubble half-way back to the center, it must be brought the rest of the way back by the leveling-screws; then $C$ will be horizontal, and if it is made so in two intersecting positions, $S$ will be vertical.

# METHODS OF ADJUSTMENT.

## I. The Compass.

**1. The Levels.** — Set up the instrument, and bring the bubbles to the center by pressure of the hands on the plate. Reverse the sights, and if the bubbles remain at the center, the levels are in adjustment. If they do not, bring each half-way back to the center by means of the screws at the ends of the level tube-case, the rest of the way by means of the plate, and repeat. Or perform this operation with one level at a time until it is nearly adjusted, then with the other, finally completing the adjustment of each and seeing that both will reverse correctly and remain in the center during an entire revolution of the plate.

**2. The Sights.** — Observe through the slits a good plumb-line and if either sight fails to range with it, make it do so by whatever means the instrument calls for, — usually, filing a little off of one side of the surface of contact of the sight with the plate.

**3. The Needle.** — If the needle will not in various positions cut opposite degrees, this adjustment is needed.

Having removed the glass top of the compass-box, with a splinter of wood bring one end of the needle to any prominent graduation, as the zero, having the eye nearly in the plane of the graduated circle, and see if the other end corresponds to the opposite division. If not, bend the center-pin (pivot) with a small wrench, about one-eight of an inch below its point, until that other end of the needle will cut the other zero. Reverse the zeros but not the ends of the needle, and, holding with the splinter the same end of the needle at the new zero, note what division the other end cuts. Bend the needle until that end cuts a division half-way back to the adjacent zero. This puts the needle's ends very approximately in line with its center. Bend the center-pin again, until the ends of the needle will cut the zeros. Repeat until perfect reversion is obtained.

Bring one end of the needle to the 90° division, and if the other end does not cut the opposite division, bend only the pivot until it does, and repeat, using alternately the line of zeros and that of the 90° divisions, until it will cut opposite degrees in any position.

## II. The Transit.

**1. The Levels.** — Adjust by reversal as in I. 1, accomplishing the reversal by means of the readings of the horizontal circle and moving the plate by means of the leveling-screws.

**2. The Needle.** — Same as I. 3.

**3. The Line of Collimation.** — Having the instrument set up and leveled on tolerably level ground, make the wires respectively horizontal and vertical by loosening the proper screws and turning the ring around until the vertical wire may be made to coincide with some known vertical line as a plumb-line, or the vertical edge of a building from two to five hundred feet distant. Make the screws tight again.

Select or locate a point from two to five hundred feet distant, clamp the plates, revolve the telescope on its axis, and locate a point on a stake on the other side of and at the same distance from the instrument as the first. Unclamp the plates and turn about the spindle until the wires can be again fixed on the first point. Clamp the plates and again revolve the telescope on its axis. If the intersection of the wires strikes the second point, the line of collimation is in adjustment. If not, the intersection of the cross-wires should be brought one-fourth of the way back to this second point by means of the pair of cross-wire screws, on the sides of the telescope, which move the vertical wire.

The operator, in loosening one of these screws and tightening the other, should remember that they have their bearings in the cross-wire ring, that a non-inverting telescope inverts the relations of the cross-wires to the object, and *vice versa*. He must therefore, in a non-inverting telescope, proceed as if apparently to move

the vertical wire in the opposite direction from that desired.

Test by repetition.

If, when this has been done, the intersection of the cross-wires is not in the center of the field of view, move the latter until they are, by means of the screws which control the eye-piece, loosening and tightening them in pairs; the movement being now direct or as it appears it should be.

Another method is to locate three points in line and all on the same side of the instrument. Then setting up over the middle point, sight to one of the end ones, and clamping the plates, revolve the telescope to sight to the other end one. If the intersection of the wires fails to strike it, move that intersection half-way to the point by means of the vertical wire, as just explained, and repeat. Then center the eye-piece as in the preceding method.

**4. The Standards.** — Select a tolerably level piece of ground in front of a tall spire, tower, or like object, that shall afford from top to base a long range in a vertical direction. Set up and level the instrument, so that it will be about as far in front of the structure as its telescope is below a good sight-point near the top. Clamp to the spindle, and, fixing the wires on the point selected, clamp the plates and lower the wires to some point found or marked at the base of the structure. (If

the ground is not very level, take the point in the face of the building, and at about the height of the ground on which the instrument stands.) Unclamp the plates, and, turning the instrument about half-way around, revolve the telescope, and again fix the wires on the lower point. Clamp the plates and raise the wires to the height of the upper point. If they cut it, the standards are in adjustment. If they do not, bring them half-way to it, by raising the right-hand end (or lowering the left) of the horizontal axis of the telescope, if the wires are to the right of the point; by raising the left (or lowering the right), if they are to the left. Most instruments have a means of making this adjustment at one end of the horizontal axis, — a movable bearing. If the instrument has no such means, file equally a little off of the feet of the higher standard. Repeat until the adjustment is perfected.

Another method is to sight to an upper point, and lowering the wires, fix a lower point at the base of the structure just as before; but, on turning the instrument about half-way around and revolving the telescope, fix the wires again on the *upper* point. Clamping, and lowering the wires to the height of the lower point, if they do not strike it, move them back towards it over that fractional part of the distance, which the height of the instrument above the ground is of double the height of the upper point above the ground. This is to be done by raising the right-hand end (or lowering the left)

of the horizontal axis, if the wires strike to the left of the point; raising the left (or lowering the right), if they come to the right. Repeat until the adjustment is perfected.

Another method (which is dependent on the accuracy of the graduation of the horizontal circle) is as follows: Setting up and leveling the instrument, clamp the plates at zero or some other convenient reading, and fix the wires on an elevated (or depressed) point of sight, clamping to the spindle. Unclamp the plates, and, turning through exactly 180°, as shown by the horizontal circle reading, clamp again. If on now raising (or lowering) the wires to the point, they cut it, the standards are in adjustment. If they do not, bring them half-way to it, by changing the height of one of the standards in such a way as to raise the right-hand end (or lower the left) of the horizontal axis, if the wires strike to the right of the point; raising the left (or lowering the right), if they come to the left.

5. *a.* **The Vertical Circle.** — (If only an arc is present, see 5 *b*.) Bring its zero into coincidence with the zero of the vernier attached to the standards, and with the telescope find or place some point or horizontal line cut by the horizontal wire and about two or three hundred feet distant. Turn the instrument about half-way around, revolve the telescope, and fixing the wires upon the point, or the horizontal wire upon the line first selected,

clamp the telescope and note if the zeros are again in coincidence. If not, loosen the screws that attach the vernier to the standards, and, moving it so as to bring its zero half-way to the vertical circle's zero, make it secure again. Repeat until no error can be detected. Instead of moving the vernier zero, the circle zero may be brought into coincidence with it, and the horizontal wire moved back over half the amount by which it has been thus displaced, provided the error is so slight as not to appreciably throw the intersection of the cross-wires out of the center of the field of view as previously adjusted. Repeat as before.

6. *a.* **Level on Telescope.** — Level the instrument carefully, and with the clamp-and-tangent movement to the horizontal axis, bring the zero of the vertical circle into coincidence with the vernier zero, and, by means of the screws at each end of the level, bring its bubble to the center, taking care not to jar the instrument out of level.

5. *b.* **Level on Telescope.** — (When the instrument has only a vertical arc and not a full circle.) On tolerably level ground stake four points in line and equidistant (about 100 feet), calling them (say) $D$, $J_1$, $E$, $J_3$, consecutively. (It is well that $D$ should not be lower than $E$, nor $J_3$ than $J_1$.) Set up the instrument at $J_1$, and direct the line of sight to a graduated rod held, as

nearly as may be, vertical on the higher of stakes $D$, $E$ (say $D$), taking a small reading $d_1$. Clamp the telescope, and turning the instrument around, take the reading $e_1$ on $E$. Set up and level the instrument at $J_3$, so that the height of the telescope shall be greater than that corresponding to the readings $d_1$, $e_1$. Unclamp the telescope and direct the line of sight so that, without changing its position, readings $e_2$, $d_2$, respectively greater than $e_1$, $d_1$, are obtained on the rod held successively on $E$ and $D$. From three times $e_2 - e_1$ take $d_2 - d_1$ and divide the result by 2. Set the target at a reading greater by this calculated amount than $d_1$, and, holding the rod on $D$, bisect the target by the line of sight and clamp the telescope. By means of the screws at the end of the level, bring its bubble to the center, and see if the line of sight still bisects the target. If it has been jarred out of position, put it back and again bring the bubble to the center. When the line of sight bisects the target and the bubble is in the center, the adjustment is complete.

A simplification of the proceeding is as follows: Instead of taking the first two readings, drive stakes so that their tops are cut by the line of sight, the telescope being clamped. Their tops are then in the same horizontal line. Making the line of sight as nearly horizontal as possible by estimation, take a reading on the nearer of these two stakes, and then, holding the rod on the farther one (target at same reading), if the line of sight

does not bisect the target, turn the telescope by the tangent-screw so that it will; repeating this until it will bisect the target held on the far stake at the same reading as on the near one. Then bring the bubble to the center by the screws at the ends of the level-tube as before.

6. *b*. **The Vertical Arc.** — With the bubble of the level on the telescope at the center, clamp the vertical arc to the axis of the telescope, with its zero in coincidence with that of the vernier, and it is in condition to correctly measure vertical angles.

## III. The Level.

1. **The Line of Collimation.** — Set the tripod firmly, remove the wye-pins from the clips, so that the telescope may be turned in its bearings, and, by means of the leveling and tangent screws, bring either of the wires into coincidence with a clearly marked edge of some object, from two to five hundred feet distant. Then turn the telescope half-way around in the wyes, so that the same wire may be compared with the edge selected. If it now coincides with the edge, it is in adjustment; if not, bring it half-way to it by moving the capstan-head screws at right angles to the wire in question, remembering the inverting property of the eye-piece. Repeat until it will reverse correctly. Then adjust the other wire in the same manner; or, if their errors are

great, make them nearly correct before exactly adjusting either.

When this has been effected, unscrew the covering of the eye-piece centering-screws, and move each pair in succession so as to bring the center of the field of view to coincide with the intersection of the cross-wires, testing the centering by revolving the telescope in the wyes, when the object should not appear to move. The screws in this case are to be moved as it *appears* they should be. Replace the covering of the screws.

**2. The Level-Bubble.** — Having the plate about horizontal, place the telescope over either pair of leveling-screws, and, clamping the instrument, remove the wye-pins and bring the bubble to the center by means of this pair of leveling-screws. Reverse the telescope end for end as to the wyes, replacing it with the level-tube immediately beneath, and note whether the bubble remains at the center. Now rotate the telescope slightly to each side, and see if this causes the bubble to move toward either end. If, after the reversal it was at the center, and remained so during the slight rotation, the level-bubble is in adjustment.

If the rotation causes it to move from its position after reversal toward either end, adjust, *by trial*, the horizontal screw so that this will not be the case. Repeat this reversal, etc., until no further adjustment of the horizontal screw is needed. Then (the telescope being in the

second position) bring the bubble half-way to the center by means of the vertical screw-nuts at one end of the level tube-case, and repeat the whole process until the bubble will remain at the center after reversal and slight rotation.

**3. The Wyes.** — Having the telescope in its normal position as to the wyes, place it over a pair of the leveling-screws and bring the bubble to the center by means of them. Turn the instrument half-way around on the spindle, and, if the bubble runs toward either end, bring it half-way back by the wye-nuts on either end of the bar and the rest of the way by the pair of leveling-screws. Then place the telescope over the other pair, and proceed in the same way, changing to each pair successively until the adjustment is completely effected for each, so that the bubble will remain at the center during an entire revolution of the telescope on the spindle.

## Peirce's Three and Four Place Tables of Logarithmic and Trigonometric Functions.

By JAMES MILLS PEIRCE, University Professor of Mathematics in Harvard University. Quarto. Cloth. Mailing Price, 45 cts.; Introduction, 40 cts.

Four-place tables require, in the long run, only half as much time as five-place tables, one-third as much time as six-place tables, and one-fourth as much as those of seven places. They are sufficient for the ordinary calculations of Surveying, Civil, Mechanical, and Mining Engineering, and Navigation; for the work of the Physical or Chemical Laboratory, and even for many computations of Astronomy. They are also especially suited to be used in teaching, as they illustrate principles as well as the larger tables, and with far less expenditure of time. The present compilation has been prepared with care, and is handsomely and clearly printed.

## Elements of the Differential Calculus.

With Numerous Examples and Applications. Designed for Use as a College Text-Book. By W. E. BYERLY, Professor of Mathematics, Harvard University. 8vo. 273 pages. Mailing Price, $2.15; Introduction, $2.00.

This book embodies the results of the author's experience in teaching the Calculus at Cornell and Harvard Universities, and is intended for a text-book, and not for an exhaustive treatise. Its peculiarities are the rigorous use of the Doctrine of Limits, as a foundation of the subject, and as preliminary to the adoption of the more direct and practically convenient infinitesimal notation and nomenclature; the early introduction of a few simple formulas and methods for integrating; a rather elaborate treatment of the use of infinitesimals in pure geometry; and the attempt to excite and keep up the interest of the student by bringing in throughout the whole book, and not merely at the end, numerous applications to practical problems in geometry and mechanics.

**James Mills Peirce**, *Prof. of Math., Harvard Univ.* (From the *Harvard Register*): In mathematics, as in other branches of study, the need is now very much felt of teaching which is general without being superficial; limited to leading topics, and yet within its limits; thorough, accurate, and practical; adapted to the communication of some degree of power, as well

as knowledge, but free from details which are important only to the specialist. Professor Byerly's Calculus appears to be designed to meet this want. . . . Such a plan leaves much room for the exercise of individual judgment; and differences of opinion will undoubtedly exist in regard to one and another point of this book. But all teachers will agree that in selection, arrangement, and treatment, it is, on the whole, in a very high degree, wise, able, marked by a true scientific spirit, and calculated to develop the same spirit in the learner. . . . The book contains, perhaps, all of the integral calculus, as well as of the differential, that is necessary to the ordinary student. And with so much of this great scientific method, every thorough student of physics, and every general scholar who feels any interest in the relations of abstract thought, and is capable of grasping a mathematical idea, ought to be familiar. One who aspires to technical learning must supplement his mastery of the elements by the study of the comprehensive theoretical treatises. . . . But he who is thoroughly acquainted with the book before us has made a long stride into a sound and practical knowledge of the subject of the calculus. He has begun to be a real analyst.

**H. A. Newton,** *Prof. of Math. in Yale Coll., New Haven:* I have looked it through with care, and find the subject very clearly and logically developed. I am strongly inclined to use it in my class next year.

**S. Hart,** *recent Prof. of Math. in Trinity Coll., Conn.:* The student can hardly fail, I think, to get from the book an exact, and, at the same time, a satisfactory explanation of the principles on which the Calculus is based; and the introduction of the simpler methods of integration, as they are needed, enables applications of those principles to be introduced in such a way as to be both interesting and instructive.

**Charles Kraus,** *Techniker, Pardubitz, Bohemia, Austria:* Indem ich den Empfang Ihres Buches dankend bestaetige muss ich Ihnen, hoch geehrter Herr gestehen, dass mich dasselbe sehr erfreut hat, da es sich durch grosse Reichhaltigkeit, besonders klare Schreibweise und vorzuegliche Behandlung des Stoffes auszeichnet, und erweist sich dieses Werk als eine bedeutende Bereicherung der mathematischen Wissenschaft.

**De Volson Wood,** *Prof. of Math., Stevens' Inst., Hoboken, N.J.:* To say, as I do, that it is a first-class work, is probably repeating what many have already said for it. I admire the rigid logical character of the work, and am gratified to see that so able a writer has shown explicitly the relation between Derivatives, Infinitesimals, and Differentials. The method of Limits is the true one on which to found the science of the calculus. The work is not only comprehensive, but no vagueness is allowed in regard to definitions or fundamental principles.

**Del Kemper,** *Prof. of Math., Hampden Sidney Coll., Va.:* My high estimate of it has been amply vindicated by its use in the class-room.

**R. H. Graves,** *Prof. of Math., Univ. of North Carolina:* I have *already* decided to use it with my next class; it suits my purpose better than any other book on the same subject with which I am acquainted.

**Edw. Brooks,** *Author of a Series of Math.:* Its statements are clear and scholarly, and its methods thoroughly analytic and in the spirit of the latest mathematical thought.

## Syllabus of a Course in Plane Trigonometry.

By W. E. BYERLY. 8vo. 8 pages. Mailing Price, 10 cts.

## Syllabus of a Course in Plane Analytical Geometry.

By W. E. BYERLY. 8vo. 12 pages. Mailing Price, 10 cts.

## Syllabus of a Course in Plane Analytic Geometry

(*Advanced Course.*) By W. E. BYERLY, Professor of Mathematics, Harvard University. 8vo. 12 pages. Mailing Price, 10 cts.

## Syllabus of a Course in Analytical Geometry of Three Dimensions.

By W. E. BYERLY. 8vo. 10 pages. Mailing Price, 10 cts.

## Syllabus of a Course on Modern Methods in Analytic Geometry.

By W. E. BYERLY. 8vo. 8 pages. Mailing Price, 10 cts.

## Syllabus of a Course in the Theory of Equations.

By W. E. BYERLY. 8vo. 8 pages. Mailing Price, 10 cts.

## Elements of the Integral Calculus.

By W. E. BYERLY, Professor of Mathematics in Harvard University. 8vo. 204 pages. Mailing Price, $2.15; Introduction, $2.00.

This volume is a sequel to the author's treatise on the Differential Calculus (see page 134), and, like that, is written as a text-book. The last chapter, however, — a Key to the Solution of Differential Equations, — may prove of service to working mathematicians.

**H. A. Newton**, *Prof. of Math., Yale Coll.:* We shall use it in my optional class next term.

**Mathematical Visitor:** The subject is presented very clearly. It is the first American treatise on the Calculus that we have seen which devotes any space to average and probability.

**Schoolmaster**, *London:* The merits of this work are as marked as those of the Differential Calculus by the same author.

**Zion's Herald:** A text-book every way worthy of the venerable University in which the author is an honored teacher. Cambridge in Massachusetts, like Cambridge in England, preserves its reputation for the breadth and strictness of its mathematical requisitions, and these form the spinal column of a liberal education.

## A Short Table of Integrals.

To accompany *BYERLY'S INTEGRAL CALCULUS.* By B. O. PEIRCE, JR., Instructor in Mathematics, Harvard University. 16 pages. Mailing Price, 10 cts. To be bound with future editions of the Calculus.

## Elements of Quaternions.

By A. S. HARDY, Ph.D., Professor of Mathematics, Dartmouth College. Crown 8vo. Cloth. 240 pages. Mailing Price, $2.15; Introduction, $2.00.

The chief aim has been to meet the wants of beginners in the class-room. The *Elements* and *Lectures* of Sir W. R. Hamilton are mines of wealth, and may be said to contain the suggestion of all that will be done in the way of Quaternion research and application: for this reason, as also on account of their diffuseness of style, they are not suitable for the purposes of elementary instruction. The same may be said of Tait's *Quaternions*, a work of great originality and comprehensiveness, in style very elegant but very concise, and so beyond the time and needs of the beginner. The *Introduction to Quaternions* by Kelland contains many exercises and examples, of which free use has been made, admirably illustrating the Quaternion spirit and method, but has been found, in the class-room, practically deficient in the explanation of the theory and conceptions which underlie these applications. The object in view has thus been to cover the *introductory* ground more thoroughly, especially in symbolic transformations, and at the same time to obtain an arrangement better adapted to the methods of instruction common in this country.

### FROM COLLEGE PROFESSORS.

**James Mills Peirce**, *Prof. of Math., Harvard Coll.:* I am much pleased with it. It seems to me to supply in a very satisfactory manner the need which has long existed of a clear, concise, well-arranged, and logically-developed introduction to this branch of Mathematics. I think Prof. Hardy has shown excellent judgment in his methods of treatment, and also in limiting himself to the exposition and illustration of the fundamental principles of his subject. It is, as it ought to be, simply a preparation for the study of the writings of Hamilton and Tait.

**Charles A. Young**, *Prof. of Astronomy, Princeton Coll.:* I find it by far the most clear and intelligible statement of the matter I have yet seen.

## Elements of the Differential and Integral Calculus.

With Examples and Applications. By J. M. TAYLOR, Professor of Mathematics in Madison University. 8vo. Cloth. 249 pp. Mailing price, $1.95; Introduction price, $1.80.

The aim of this treatise is to present simply and concisely the fundamental problems of the Calculus, their solution, and more common applications. Its axiomatic datum is that the change of a variable, when not uniform, may be conceived as becoming uniform at any value of the variable.

It employs the conception of rates, which affords finite differentials, and also the simplest and most natural view of the problem of the Differential Calculus. This problem of *finding the relative rates of change of related variables* is afterwards reduced to that of finding the limit of the ratio of their simultaneous increments; and, in a final chapter, the latter problem is solved by the principles of infinitesimals.

Many theorems are proved both by the method of rates and that of limits, and thus each is made to throw light upon the other. The chapter on differentiation is followed by one on direct integration and its more important applications. Throughout the work there are numerous practical problems in Geometry and Mechanics, which serve to exhibit the power and use of the science, and to excite and keep alive the interest of the student.

Judging from the author's experience in teaching the subject, it is believed that this elementary treatise so sets forth and illustrates the highly practical nature of the Calculus, as to awaken a lively interest in many readers to whom a more abstract method of treatment would be distasteful.

**Oren Root, Jr.**, *Prof. of Math., Hamilton Coll., N.Y.:* In reading the manuscript I was impressed by the clearness of definition and demonstration, the pertinence of illustration, and the happy union of exclusion and condensation. It seems to me most admirably suited for use in college classes. I prove my regard by adopting this as our text-book on the calculus.

**C. M. Charrappin, S.J.**, *St. Louis Univ.:* I have given the book a thorough examination, and I am satisfied that it is the best work on the subject I have seen. I mean the best work for what it was intended,—a text-book. I would like very much to introduce it in the University. (*Jan.* 12, 1885.)

**J. G. Fox**, *Prof. of Civil Eng., Lafayette Coll., Easton, Pa.:* It has some very good points in its favor, such as, the arrangement of the subject-matter, the "numerous practical problems," etc. (*Feb.* 21, 1885.)

**J. Howard Gore**, *Prof. of Math., Columbian Coll., Washington, D.C.:* From a careful inspection I think that in very many respects it is a marked improvement on the various works on calculus now in use. I have always thought that integral and differential calculus should be studied at the same time. This is not feasible except when the author arranged the subject-matter with that plan in view, as in this book. At present I regard with favor the introduction of this work in my classes next session. (*Jan.* 9, 1885.)

**C. H. Judson**, *Prof. of Math., Furman Univ., Greenville, S.C.:* I find it to be one of the most accurate, logical, and carefully prepared text-books that I have met with. I believe there is no better text-book for teachers who adopt the theory of "Rates" as the basis of the calculus. (*Dec.* 30, 1884.)

**O. C. Gray**, *Prof. of Math., Ark. Indus. Univ.:* Thus far, I am very much pleased with it, particularly in the fact that chapters on Differentiation and Integration immediately follow each other. Such an arrangement was needed. (*Jan.* 6, 1885.)

**F. L. Morse**, *Prof. of Math., Hanover Coll., Ind.:* The matter is certainly admirably chosen, and the arrangement natural and unique. The Integral Calculus is placed in proper order, and the practical application to Mechanics will do much to clear away the mysteries of which the student often justly complains. (*Dec.* 25, 1884.)

**London Schoolmaster:** This easy but comprehensive treatise on the calculus differs in its methods from similar text-books produced on this side of the Atlantic. Altogether, considering the extent of the ground it covers, it is one of the easiest and clearest text-books on the calculus we know.

**Boston Advertiser:** It reflects a high measure of credit upon the author. He certainly has succeeded to a degree that may well insure for the present volume extended use as a text-book in our colleges. He has shown a thorough, comprehensive grasp of his subject, and has brought this to bear with singular force in his pointed definitions, and in the clear reasoning of his demonstrations.

**The Nation**, *New York:* It has two marked characteristics. In the first place, it is evidently a most carefully written book. There is nothing vague or slipshod in it. Nearly every sentence, certainly every theorem, seems to have been constructed with a strenuous effort to give it clearness and precision. This constant attention to the form of expression has enabled the author to be concise without becoming obscure. We are acquainted with no text-book of the calculus which compresses so much matter into so few pages, and at the same time leaves the impression that all that is necessary has been said. In the second place, the number of carefully selected examples, both of those worked out in full in illustration of the text, and of those left for the student to work out for himself, is extraordinary. From this point of view those teachers and pupils who are accustomed to or prefer a different text-book, would still do well to provide themselves with this, regarding it merely as a collection of examples and without any reference to the text.

## Metrical Geometry: An Elementary Treatise on Mensuration.

By GEORGE BRUCE HALSTED, Ph.D., Prof. Mathematics, University of Texas, Austin. 12mo. Cloth. 246 pages. Mailing price, $1.10; Introduction, $1.00.

This work applies new principles and methods to simplify the measurement of lengths, angles, areas, and volumes. It is strictly demonstrative, but uses no Trigonometry, and is adapted to be taken up in connection with, or following any elementary Geometry. It treats of accessible and inaccessible straight lines, and of their interdependence when in triangles, circles, etc.; also gives a more rigid rectification of the circumference, etc. It introduces the natural unit of angle, and deduces the ordinary and circular measure. Enlisting the auxiliary powers which modern geometers have recognized in notation, it binds up each theorem also in a self-explanatory formula, and this throughout the whole book on a system which renders confusion impossible, and surprisingly facilitates acquirement, as has been tested with very large classes in Princeton College. In addition to all the common propositions about areas, a new method, applicable to any polygon, is introduced, which so simplifies and shortens all calculations, that it is destined to be universally adopted in surveying, etc. In addition to the circle, sector, segment, zone, annulus, etc., the parabola and ellipse are measured; and besides the common broken and curved surfaces, the theorems of Pappus are demonstrated. Especial mention should be made of the treatment of solid angles, which is original, introducing for the first time, we think, the natural unit of solid angle, and making spherics easy. For solids, a single informing idea is fixed upon of such fecundity as to place within the reach of children results heretofore only given by Integral Calculus. Throughout, a hundred illustrative examples are worked out, and at the end are five hundred carefully arranged and indexed exercises, using the metric system.

**OPINIONS.**

**Simon Newcomb,** *Nautical Almanac Office, Washington, D.C.:* I am much interested in your Mensuration, and wish I had seen it in time to have some exercises suggested by it put into my Geometry. (*Sept.* 8, 1881.)

**Alexander MacFarlane,** *Examiner in Mathematics to the University of Edinburgh, Scotland:* The method, figures, and examples appear excellent, and I anticipate much benefit from its minute perusal.

## *Elementary Co-ordinate Geometry.*

By W. B. SMITH, Professor of Physics, Missouri State University. 12mo. Cloth. 312 pp. Mailing price, $2.15; for Introduction, $2.00.

While in the study of Analytic Geometry either gain of knowledge or culture of mind may be sought, the latter object alone can justify placing it in a college curriculum. Yet the subject may be so pursued as to be of no great educational value. Mere calculation, or the solution of problems by algebraic processes, is a very inferior discipline of reason. Even geometry is not the best discipline. In all thinking the real difficulty lies in forming clear notions of things. In doing this all the higher faculties are brought into play. It is this formation of concepts, therefore, that is the essential part of mental training. He who forms them clearly and accurately may be safely trusted to put them together correctly. Nearly every seeming mistake in reasoning is really a mistake in conception.

Such considerations have guided the composition of this book. Concepts have been introduced in abundance, and the proofs made to hinge directly upon them. Treated in this way the subject seems adapted, as hardly any other, to develop the power of thought.

Some of the special features of the work are: —

1. Its SIZE is such it can be mastered in the time generally allowed.

2. The SCOPE is far wider than in any other American work.

3. The combination of *small size* and *large scope* has been secured through SUPERIOR METHODS, — *modern, direct, and rapid.*

4. Conspicuous among such methods is that of DETERMINANTS, here presented, by the union of theory and practice, in its real power and beauty.

5. Confusion is shut out by a consistent and self-explaining NOTATION.

6. The ORDER OF DEVELOPMENT is *natural*, and leads without break or turn from the simplest to the most complex. The method is *heuristic.*

7. The student's grasp is strengthened by numerous EXERCISES.

8. The work has been TESTED at every point IN THE CLASS-ROOM.

## Determinants.

The Theory of Determinants: an Elementary Treatise. By PAUL H. HANUS, B.S., Professor of Mathematics in the University of Colorado. 8vo. Cloth. 000 pages. Mailing price, $0.00; for Introduction, $0.00.

This is a text-book for the use of students in colleges and technical schools. The need of an American work on determinants has long been felt by all teachers and students who have extended their reading beyond the elements of mathematics. The importance of the subject is no longer overlooked. The shortness and elegance imparted to many otherwise tedious processes, by the introduction of determinants, recommend their use even in the more elementary branches, while the advanced student cannot dispense with a knowledge of these valuable instruments of research. Moreover, determinants are employed by all modern writers.

This book is written especially for those who have had no previous knowledge of the subject, and is therefore adapted to self-instruction as well as to the needs of the class-room. To this end the subject is at first presented in a very simple manner. As the reader advances, less and less attention is given to details. Throughout the entire work it is the constant aim to arouse and enliven the reader's interest by first showing how the various concepts have arisen naturally, and by giving such applications as can be presented without exceeding the limits of the treatise. The work is sufficiently comprehensive to enable the student that has mastered the volume to use the determinant notation with ease, and to pursue his further reading in the modern higher algebra with pleasure and profit.

In Chapter I. the evolution of a theory of determinants is touched upon, and it is shown how determinants are produced in the process of eliminating the variables from systems of simple equations with some further preliminary notions and definitions.

In Chapter II. the most important properties of determinants are discussed. Numerous examples serve to fix and exemplify the principles deduced.

Chapter III. comprises half the entire volume. It is the design of this chapter to familiarize the reader with the most important special forms that occur in application, and to enable him to realize the practical usefulness of determinants as instruments of research.

[*Ready June* 1.